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The role of the pion pair term in the theory of the weak axial meson exchange currents

B. Mosconi[1,2,a], P. Ricci[2,b], and E. Truhlík[3,c]

[1] Dipartimento di Fisica, Università di Firenze, I-50019, Sesto Fiorentino (Firenze), Italy
[2] Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, I-50019, Sesto Fiorentino (Firenze), Italy
[3] Institute of Nuclear Physics, Academy of Sciences of the Czech Republic, CZ-250 68 Řež, Czech Republic

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Abstract. The structure of the weak axial pion exchange current is discussed in various models. It is shown how the interplay of the chiral invariance and the double-counting problem restricts uniquely the form of the pion potential term, to the case when the nuclear dynamics is described by the Schrödinger equation with static nucleon-nucleon potential.

PACS. 11.40.Ha Partially conserved axial-vector currents – 25.30.-c Lepton-induced reactions

1 Introduction

The semileptonic weak nuclear interaction has been studied for half a century. The basic cornerstones of this field of research are i) the chiral symmetry, ii) the conserved vector current and iii) the partial conservation of the axial current (PCAC). In the formulation [1], the PCAC reads

$$q_\mu \langle \Psi_f | j^a_{5\mu}(q) | \Psi_i \rangle \,=\, i f_\pi m^2_\pi \Delta^\pi_F(q^2) \langle \Psi_f | m^a_\pi(q) | \Psi_i \rangle \,, \quad (1)$$

where $j^a_{5\mu}(q)$ is the total weak axial isovector current, $m^a_\pi(q)$ is the total pion source (the pion production/absorption amplitude) and $|\Psi_{i,f}\rangle$ is the wave function describing the initial (i) or final (f) nuclear state.

It has been recognized [2] in studying the triton beta decay,

$$^3\mathrm{H} \,\rightarrow\, ^3\mathrm{He} \,+\, e^- \,+\, \bar{\nu} \,, \quad (2)$$

that in addition to the one-nucleon current, the effect of the space component of weak axial exchange current (WAEC) enhances the Gamow-Teller matrix element that is to be compared to the one extracted from the data. This suggests that the current $j^a_{5\mu}(q)$ can be understood for the system of A nucleons as the sum of the one- and two-nucleon components,

$$j^a_{5\mu}(q) \,=\, \sum^A_{i=1} j^a_{5\mu}(1,i,q_i) \,+\, \sum^A_{i<j} j^a_{5\mu}(2,ij,q) \,. \quad (3)$$

[a] e-mail: mosconi@fi.infn.it
[b] e-mail: ricci@fi.infn.it
[c] e-mail: truhlik@ujf.cas.cz

Let us describe the nuclear system by the Schrödinger equation

$$H|\Psi\rangle \,=\, E|\Psi\rangle \,, \quad (4)$$

with the Hamiltonian H,

$$H \,=\, T \,+\, V \,, \quad (5)$$

where T is the kinetic energy and V is the nuclear potential describing the interaction between nucleon pairs. Taking for simplicity $A = 2$, we obtain from eq. (1) in the operator form and from eqs. (3) and (5) the following set of equations for the one- and two-nucleon components of the total axial current

$$\vec{q}_i \cdot \vec{j}^{\,a}_5(1,\vec{q}_i) = [\, T_i \,,\, \rho^a_5(1,\vec{q}_i)\,] \,+\, i f_\pi m^2_\pi \Delta^\pi_F(q^2)\, m^a_\pi(1,\vec{q}_i) \,, \quad i = 1,2 \,, \quad (6)$$

$$\vec{q} \cdot \vec{j}^{\,a}_5(2,\vec{q}) = [\, T_1 + T_2 \,,\, \rho^a_5(2,\vec{q})\,] + ([\, V \,,\, \rho^a_5(1,\vec{q})\,] + (1 \leftrightarrow 2)) + i f_\pi m^2_\pi \Delta^\pi_F(q^2)\, m^a_\pi(2,\vec{q}) \,. \quad (7)$$

In eq. (7), we neglected $\rho^a_5(2,\vec{q})$ in the second commutator on the right-hand side. If the WAEC is constructed so that it satisfies eq. (7), then the matrix element of the total current, sandwiched between solutions of the Schrödinger equation (4), satisfies the PCAC (1).

It is known from the dimensional analysis [3], that the space component of the WAEC $\vec{j}^{\,a}_5(2,\vec{q})$ is of the order $\mathcal{O}(1/M^3)$ (M is the nucleon mass). Being of a relativistic origin, it is model dependent. This component of the WAEC was derived by several authors in various models. In the standard nuclear physics approach the model systems of strongly interacting particles contain various particles, such as baryons N, $\Delta(1232)$, pions and heavy mesons [4–10]. On the other hand, in effective field theories, one uses Lagrangians with the heavy particle degrees of freedom integrated out and preserving nucleon, delta and pion [11] or nucleon and pion [12,13] or only nucleon [14,15] degrees of freedom.

Accepting the chiral symmetry as the basic symmetry governing the nuclear dynamics, it is expected that the WAEC of the pion range, constructed within approaches respecting this symmetry and in conjunction with the given nuclear equation of motion, should exhibit model independence. On the other hand, checking the weak axial pion exchange currents, constructed in [5–10,13,15], one concludes that the situation is not transparent. Let us discuss various approaches in more detail.

Let us first classify, in general, the WAEC of a given range into potential and non-potential currents. In analogy with the electromagnetic sector, the potential WAEC is such that it satisfies the part of eq. (7) containing the commutator $[\, V\, ,\, \rho^a_5(1,\vec{q}\,)\,]$. As in the case of the electromagnetic interaction, the pair term is one of the exchange currents that belong to the potential current. It is obtained by the non-relativistic reduction of the negative frequency part of the nucleon Born term. Besides, other potential currents can appear. Then the total potential WAEC is defined as the sum of all potential terms of a given range.

The approach of ref. [9] in constructing the WAEC is the only one that is not based on chiral Lagrangians. It uses the relativistic nucleon Born terms and the WAEC is obtained by embedding the nuclear potential into the negative-frequency part of these terms, thus directly connecting the potentials and the WAEC. It follows that this current is not chiral since it is not based on any chiral group. The pion pair term is obtained using the pseudovector (PV) πNN coupling[1], which could be considered as an argument that the global chiral invariance is respected. However, any model is chiral invariant only if the resulting current does not depend on the choice of the πNN coupling, which is not the case of ref. [9]. As discussed in the last paragraph of sect. 2 of ref. [9], this construction does not tolerate the pseudoscalar (PS) πNN coupling, since it provides the weak pion production amplitude that is at variance with the current algebra prediction. However, it was overlooked in [9] that the chiral model [4] with the PS πNN coupling does provide the correct weak pion production amplitude. In other words, one should use chiral models and not simple πNN couplings.

In ref. [6], the WAEC is derived within the extended S-matrix method [16], using the chiral Lagrangian model with the PV πNN coupling [4] . The resulting potential current is of the order $\mathcal{O}(1/M^3)$ and is given by the difference of the nucleon Born term and the first Born iteration. In ref. [7], the same potential current is obtained from the chiral model [4] with the PS πNN coupling. In this case, besides the pair term, the PCAC constraint term contributes.

[1] Being of the order $\mathcal{O}(1/M^5)$, it is negligible.

The pion pair term of refs. [8,17] is derived from the PS πNN coupling that is not chiral invariant, as it was correctly noted in ref. [18].

In ref. [13], the WAEC is derived within the heavy baryon chiral perturbation theory (HBChPT) approach, but the pion pair term is considered as fully reducible and therefore omitted.

Moreover, it is not clear from refs. [8,9,13] that the constructed WAEC of the pion range satisfies a particular form of the PCAC in conjunction with a specific nuclear equation of motion[2]. In other words, the problem of double counting is overlooked. As it will become clear later, these currents do not satisfy the PCAC as stated in eq. (1), if used in standard nuclear physics calculations, based on the Schrödinger equation and static nuclear potentials.

Here we shall discuss the role of the weak axial pion pair term in fulfilling eq. (7) in conjunction with the Schrödinger equation and the static nuclear potential. Simultaneously, we shall consider the problem of double counting. We shall construct the pion pair term and the related potential current in two models. In sect. 2.1, we start from the Lagrangian of the πN system used in the chiral perturbation theory, from which we construct in the leading order (tree approximation) the WAEC of the pion range. We explicitly show how the potential and non-potential parts interplay with other components entering eq. (7) so that the continuity equation is satisfied. The resulting potential term is the same as the one derived earlier in [4–7] from the hard pion Lagrangian of the $N\Delta\pi\rho a_1$ system. In sect. 2.2, we derive the potential term in the leading order of the HBChPT approach. We show that the obtained current is the same as the one derived in sect. 2.1. We compare the space component of the long-range part of the WAEC computed in various models in sect. 2.3, where we also calculate the effect of the potential term in the deuteron weak disintegration by low-energy neutrinos in the neutral-current channel. In sect. 3, we summarize our results.

2 The pion pair term and the nuclear PCAC

In constructing the weak axial potential pion exchange operator we start from the set of relativistic Feynman amplitudes satisfying the PCAC equation. Generally, these amplitudes are not yet the nuclear exchange currents, because of the double-counting problem: the presence of the pair term in the exchange current operator is related to the equation, describing the nuclear states. If the nucleon propagator in the first Born iteration is the full relativistic one, then this iteration is equal to the nucleon Born term and the exchange currents do not contain any pair term, in order to avoid the double counting. This is the case of the axial currents constructed in conjunction with the Bethe-Salpeter equation [19]. In this case, the nucleon Born term is fully reducible. In the case of the Schrödinger

[2] According to ref. [17], the axial current is not supposed to satisfy any continuity equation, in contrast to the electromagnetic current.

equation, the nucleon Born term is not fully reducible. The propagator of the first Born iteration contains only the positive frequencies and usually, the nuclear potential is the static one. Then the negative-frequency part is not the only one to contribute to the exchange current operator from the nucleon Born term. If the Feynman amplitudes are constructed using the chiral model with the PV coupling, the positive-frequency part of the nucleon Born term does not coincide with the first Born iteration and the difference should be calculated. Then the resulting potential current is equal to this difference, since the negative frequency part of the nucleon Born term (the pair term) is suppressed by a factor $\approx 1/M^2$ and therefore, negligible.

Let us note that the method of the construction of the nuclear WAEC [5–7] we use here was considered earlier [16] also for the construction of the electromagnetic exchange currents. The resulting exchange currents, $j_\mu(2,q)$, containing the leading relativistic corrections in both the space and time components, satisfy the current conservation constraint

$$q_\mu j_\mu(2) \,=\, ([\,V\,,\,\rho(1,\vec{q}\,)\,] + (1 \leftrightarrow 2))\,. \tag{8}$$

These currents coincide with the exchange currents derived within the framework of the transformation method as it was shown in ref. [20].

Below we shall use two model Lagrangians of the πN system that also include the external electroweak fields aiming to demonstrate the appearance of the potential term of the same order in $1/M$ as other axial pion-exchange currents, and to show that its presence is required by the PCAC hypothesis. The first model Lagrangian is the basic Lagrangian of the chiral perturbation theory [21]. The second model Lagrangian is that of the HBChPT used in [12] to construct the WAEC. We construct the currents in the leading order only, since the higher-order corrections cannot change our conclusions.

2.1 The weak axial pion pair term within the formalism of the chiral perturbation theory

We start from the Lagrangian of the πN system [21–23]

$$\begin{aligned}\mathcal{L}_{\pi N} &= -\bar{N}\,\gamma_\mu\,(\partial_\mu - i\bar{\alpha}_{\mu\,\|})\,N \\ &\quad -M\,\bar{N}N \,+\, ig_A\bar{N}\,\gamma_\mu\gamma_5\,\bar{\alpha}_{\mu\,\perp}\,N\,, \qquad (9)\\ \bar{\alpha}_\mu &= -i[\partial_\mu\xi(\pi)]\xi^+ \,-\, e\xi\,(\mathcal{V}_\mu + \mathcal{A}_\mu)\,\xi^+ \\ &\equiv \bar{\alpha}_{\mu\,\|} \,+\, \bar{\alpha}_{\mu\,\perp}\,. \qquad (10)\end{aligned}$$

Here $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are the external vector and axial vector fields, and

$$\begin{aligned}&\bar{\alpha}_{\mu\,\|} \,=\, (2trS^a\bar{\alpha}_\mu)\,S^a\,, \quad \bar{\alpha}_{\mu\,\perp} \,=\, (2trX^a\bar{\alpha}_\mu)\,X^a\,,\\ &\xi(\pi) = \exp(-i\pi(x)/f_\pi)\,,\\ &\pi(x) \,=\, \sum_a \pi^a(x)X^a\,, \qquad (11)\\ &X^a \,=\, \frac{\tau^a}{2}\gamma_5\,, \qquad S^a \,=\, \frac{\tau^a}{2}\,. \qquad (12)\end{aligned}$$

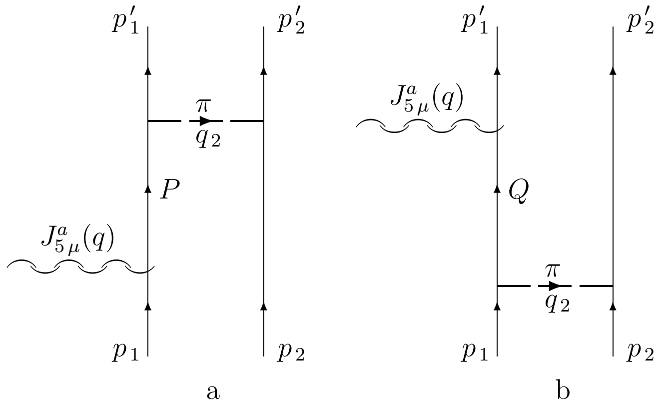

Fig. 1. The weak axial nucleon Born term of the pion range.

The current of our interest is presented in fig. 1. In order to derive the contribution of this current to the space component of the WAEC, we need to extract from the Lagrangian (9) the lowest-order vertices

$$\varDelta\mathcal{L}_{\pi N} \,=\, -ig_A\,\bar{N}\,\gamma_\mu\gamma_5\,\frac{\vec{\tau}}{2}\,N\cdot\vec{\mathcal{A}}_\mu \,-\, i\frac{g_A}{2f_\pi}\,\bar{N}\,\gamma_\mu\gamma_5\,\vec{\tau}\,N\cdot\partial_\mu\vec{\pi}\,. \tag{13}$$

The Feynman amplitude reads

$$\begin{aligned}J^a_{5\mu}(pv) \,=\, &-\bar{u}(p_1')\Big[\hat{\mathcal{O}}^\pi_1(-q_2)S_F(P)\,\hat{J}_{5\mu}(1,q)\frac{1}{2}(a^+ - a^-)\\ &+\hat{J}_{5\mu}(1,q)\,S_F(Q)\hat{\mathcal{O}}^\pi_1(-q_2)\frac{1}{2}(a^+ + a^-)\Big]\\ &\times u(p_1)\varDelta^\pi_F(q_2^2)\bar{u}(p_2')\hat{\mathcal{O}}^\pi_2(q_2)u(p_2) \,+\, (1 \leftrightarrow 2)\,,\end{aligned} \tag{14}$$

where

$$\hat{\mathcal{O}}^\pi(q_2) \,=\, \frac{f_{\pi NN}}{m_\pi}\,\not{q}_2\gamma_5\,, \quad a^\pm \,=\, \frac{1}{2}\,[\tau_1^a,\,\tau_1^n]_\pm\,\tau_2^n\,, \tag{15}$$

and we consider only the contact part of the one-nucleon current

$$\hat{J}_{5\mu}(1,c) \,=\, -ig_A\,\gamma_\mu\gamma_5\,. \tag{16}$$

In calculating the contribution of the amplitude $J^a_{5\mu}(pv)$ to the exchange currents, one splits the nucleon propagator into the positive- and negative-frequency parts and the non-relativistic reduction is made. As already discussed in sect. 1, the contribution to the space component of the negative-frequency part of the Feynman amplitude is of the nominal order $\mathcal{O}(1/M^5)$ and therefore, negligibly small. In the extended S-matrix method[3] [16,24], first the positive-frequency part of the amplitude $J^a_{5\mu}(pv)$ is

[3] The same procedure has recently been applied in the study of the e-d scattering in ref. [25].

written as

$$J^{a(+)}_{5\mu}(pv) \;=\; \frac{f_{\pi NN}}{m_\pi}\bar u(p_1')\Big[\;(\vec q_2\cdot\vec\gamma + iq_{20}\gamma_4)\,\gamma_5\frac{1}{P_0 - E(\vec P)}$$
$$\times u(P)\bar u(P)\,\hat J_{5\mu}(1,q)\,\frac{1}{2}(a^+ - a^-)$$
$$+\hat J_{5\mu}(1,q)\frac{1}{Q_0 - E(\vec Q\,)}u(Q)\bar u(Q)$$
$$\times\,(\vec q_2\cdot\vec\gamma + iq_{20}\gamma_4)\,\gamma_5\,\frac{1}{2}(a^+ + a^-)\Big]u(p_1)$$
$$\times\,\Delta^\pi_F(q_2)\bar u(p_2')\hat{\mathcal O}^\pi_2(q_2)u(p_2) + (1\,\leftrightarrow\,2)\,, \tag{17}$$

For the graph fig. 1a there holds

$$q_{20} = P_0 - p_{10}' \,=\, P_0 - E(\vec P) \,+\, E(\vec P) - p_{10}'$$
$$\equiv P_0 - E(\vec P) \,+\, q^{st}_{20}\,. \tag{18}$$

A similar equation holds for the graph fig. 1b. Then one obtains

$$J^{a(+)}_{5\mu}(pv) \;=\; J^{a(+)}_{5\mu}(ps) \,+\, \Delta J^a_{5\mu}(pv)\,. \tag{19}$$

Here $J^{a(+)}_{5\mu}(ps)$ is the positive frequency part of the nucleon Born term obtained using the static PS πNN coupling. It is the current containing a contribution from the potential, since it coincides with the first Born iteration of the Lippmann-Schwinger equation, if the static one-pion exchange potential is used. In order to avoid double counting, the contribution from such a graph is not included in the exchange current since it is reducible.

The current $\Delta J^a_{5\mu}(pv)$ arises from the contact interaction

$$\Delta J^a_{5\mu}(pv) \;=\; i\frac{f_{\pi NN}}{m_\pi}\,\bar u(p_1')\,\Big[\gamma_4\gamma_5\,u(P)\bar u(P)\,\hat J_{5\mu}(1,q)$$
$$\times\,\frac{1}{2}(a^+ - a^-) - \hat J_{5\mu}(1,q)\,u(Q)\bar u(Q)\gamma_4\gamma_5\,\frac{1}{2}(a^+ + a^-)\Big]$$
$$\times\,u(p_1)\Delta^\pi_F(q_2)\bar u(p_2')\hat{\mathcal O}^\pi_2(ps)u(p_2) + (1\,\leftrightarrow\,2)\,, \tag{20}$$

where $\hat{\mathcal O}^\pi_2(ps) = ig_{\pi NN}\gamma_5$. The non-relativistic reduction of the space component of the current (20) yields

$$\Delta\vec j^{\,a}_5(pv) \;=\; g_A\frac{g^2_{\pi NN}}{(2M)^3}\Big[\,\Big(\vec q + i\vec\sigma_1\times\vec P_1\Big)\;{\tau_2}^a$$
$$+\,\Big(i\vec P_1 \,-\, \vec\sigma_1\times\vec q\Big)\;(\vec\tau_1\times\vec\tau_2)^a\Big]$$
$$\times\,\Delta^\pi_F({\vec q_2}^{\,2})(\vec\sigma_2\cdot\vec q_2) + (1\,\leftrightarrow\,2)\,, \tag{21}$$

where $\vec P_1 = \vec p_1 + {\vec p_1}'$. This current coincides with the potential term derived earlier [6] from the hard-pion Lagrangian with the PV πNN coupling [4,5] and it contributes to the space component of the WAEC in the same leading order in $1/M$ as other pion exchange currents.

The well-known Foldy-Dyson unitary transformation of the nucleon field [26,27] can be used in the Lagrangian (9) to obtain the PS πNN coupling

$$N \;=\; \exp\left[-i\,\frac{g_A}{2f_\pi}\,\gamma_5\,(\vec\tau\cdot\vec\pi)\right]\,N'\,. \tag{22}$$

In this case, together with the nucleon Born term $J^a_{5\mu}(ps)$ a contact amplitude $J^a_{5\mu}(\mathrm{PCAC})$, called the PCAC constraint term, appears. For these amplitudes, the following equation holds:

$$J^a_{5\mu}(pv) \;=\; J^a_{5\mu}(ps) \,+\, J^a_{5\mu}(\mathrm{PCAC})\,. \tag{23}$$

It is clear that the resulting amplitude does not depend on the nature of the πNN coupling. This is due to the validity of the powerful representation independence (equivalence) theorem [28–30].

In order to extract the nuclear WAEC from the relativistic amplitudes in this case, the reducible part of the nucleon Born amplitude $J^a_{5\mu}(ps)$ is isolated. This is the positive-frequency part $J^{a\,(+)}_{5\mu}(ps)$. Then from eqs. (19) and (23) we get

$$\Delta J^a_{5\mu}(pv) \;=\; J^{a\,(-)}_{5\mu}(ps) \,+\, J^a_{5\mu}(\mathrm{PCAC})\,, \tag{24}$$

where $J^{a\,(-)}_{5\mu}(ps)$ is the negative-frequency part of the nucleon Born term obtained with the PS πNN coupling. Explicitly, one has for the space component of the nuclear current, given by the right-hand side of eq. (24)

$$\vec j^{\,a}_5(ps) = g_A\frac{g^2_{\pi NN}}{(2M)^3}$$
$$\times\Big[\,\Big(\vec q \,+\, i\vec\sigma_1\times\vec P_1\Big)\;{\tau_2}^a - (\vec\sigma_1\times\vec q_2)\;(\vec\tau_1\times\vec\tau_2)^a\,\Big]$$
$$\times\,\Delta^\pi_F({\vec q_2}^{\,2})(\vec\sigma_2\cdot\vec q_2) \,+\, (1\,\leftrightarrow\,2)\,, \tag{25}$$

and

$$\vec j^{\,a}_5(\mathrm{PCAC}) = g_A\frac{g^2_{\pi NN}}{(2M)^3}\;\Big[\,i\vec P_1 \,-\, (\vec\sigma_1\times\vec q_1)\Big]\;(\vec\tau_1\times\vec\tau_2)^a$$
$$\times\,\Delta^\pi_F({\vec q_2}^{\,2})(\vec\sigma_2\cdot\vec q_2) \,+\, (1\,\leftrightarrow\,2)\,. \tag{26}$$

So in the chiral model with the PS πNN coupling, the potential current is obtained as the sum of the negative-frequency part of the nucleon Born term and the PCAC constraint term. This leads to the equality given by eq. (21).

The derivation of the WAEC from the hard pion Lagrangian with the PS πNN coupling was carried out earlier [5,7] with the following consequences:

i) In a chiral invariant model with the PS πNN coupling, additional potential term arises, that makes the resulting current equivalent to the current of the chiral model with the PV πNN coupling. It follows the necessity of constructing the WAEC within the chiral models and not simply in terms of πNN couplings.

ii) In order to avoid double counting, the reducible part of the potential current should be removed, since it is taken into account already at the level of the impulse approximation calculations. This procedure depends on the nuclear equation of motion used for the description of nuclear states. Here the calculation is carried out for the Schrödinger equation and static one-pion exchange potentials. In our opinion, the problem of double counting was omitted in [8,9,12]. Since the potential term is absent in [12], those currents should be used in conjunction with the Bethe-Salpeter equation, because, as discussed in [19, 31], the WAEC does not contain the contribution from the nucleon Born term in this case. On the other hand, these currents [12] are used at present in nuclear physics calculations with wave functions derived with the Schrödinger equation [18,32,33].

Let us now discuss the continuity equation (1) for our current. It can be shown that the nucleon Born term due to the contact part of the one-body current (16) of our model satisfies the continuity equation

$$q_\mu J^a_{5\mu,\pi}(B,c) \,=\, i f_\pi\, M^a_\pi(B)\,, \tag{27}$$

where $M^a_\pi(B)$ is the pion Born absorption amplitude given by the graph of fig. 1 with the pion line instead of the weak-interaction wavy line inserted. Then the related nuclear continuity equation for the nuclear current reads

$$q_\mu j^a_{5\mu,\,\pi}(B,c) \,=\, i f_\pi m^a_\pi(2) \,+\, ([V_\pi,\rho^a_5(1,c)] \,+\, (1 \leftrightarrow 2))\,. \tag{28}$$

Here the space part of the current $j^a_{5\mu,\,\pi}(B,c)$ is given by eq. (21) with the divergence

$$\vec{q}\cdot\Delta\vec{j}^{\,a}_5(pv) = \frac{g_A^3}{8Mf_\pi^2}\Bigg\{\left[\vec{q}^{\,2} \,+\, i(\vec{q}\cdot\vec{\sigma}_1\times\vec{P}_1)\right]\tau_2^a + i(\vec{q}\cdot\vec{P}_1)\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Bigg\}\Delta^\pi_F(\vec{q}_2^{\,2})(\vec{\sigma}_2\cdot\vec{q}_2) + (1 \leftrightarrow 2)\,, \tag{29}$$

while it holds for the time component that

$$q_0\Delta j^a_{50}(pv) \,\approx\, \mathcal{O}(1/M^5)\,. \tag{30}$$

The pion absorption amplitude is obtained by the same method used above for the derivation of the current $\Delta\vec{j}_5(pv)$. Besides the contribution $m^a_\pi(2,\mathrm{ver})$ from the energy dependence of the πNN vertex of the internal pion, the contribution $m^a_\pi(2,\mathrm{ext})$ from the energy dependence of the πNN vertex of the external pion arises with the result

$$i f_\pi m^a_\pi(2,\mathrm{ver}) \,=\, \vec{q}\cdot\Delta\vec{j}^{\,a}_5(pv)\,, \tag{31}$$

$$i f_\pi m^a_\pi(2,\mathrm{ext}) \,=\, \frac{g_A^3}{8Mf_\pi^2}\Bigg\{\left[\vec{q}_2^{\,2} \,-\, i(\vec{q}_2\cdot\vec{\sigma}_1\times\vec{P}_1)\right]\tau_2^a + i(\vec{q}_2\cdot\vec{P}_1)\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Bigg\}\Delta^\pi_F(\vec{q}_2^{\,2})(\vec{\sigma}_2\cdot\vec{q}_2) + (1 \leftrightarrow 2)\,. \tag{32}$$

It is straightforward to obtain that the commutator of the static one-pion exchange potential and the one-nucleon axial charge density

$$\rho^a_5(1,c)_i \,=\, \frac{g_A}{2M}(\vec{\sigma}_i\cdot\vec{P}_i)\frac{\tau^a}{2}\,, \tag{33}$$

is given by

$$([V_\pi,\rho^a_5(1,c)] \,+\, (1 \leftrightarrow 2)) \,=\, -i f_\pi m^a_\pi(2,\mathrm{ext})\,. \tag{34}$$

The continuity equation (28) which is in the leading order in $1/M$ of the form

$$\vec{q}\cdot\Delta\vec{j}^{\,a}_5(pv) = i f_\pi\left[m^a_\pi(2,\mathrm{ver}) \,+\, m^a_\pi(2,\mathrm{ext})\right] + ([V_\pi,\rho^a_5(1,c)] \,+\, (1 \leftrightarrow 2))\,, \tag{35}$$

is satisfied exactly. This is established from eqs. (29)-(34). The contact term $\Delta j^a_{5\mu}(pv)$ is related to the part of the continuity equation, containing the potential and can be called as the true potential current.

Besides the nucleon Born term, our model Lagrangian contains a $\mathcal{A}\pi NN$ vertex

$$\Delta\mathcal{L}_{\mathcal{A}\pi NN} \,=\, -\frac{i}{2f_\pi}\bar{N}\,\gamma_\mu\,\vec{\tau}\,N\cdot\left(\vec{\pi}\times\vec{\mathcal{A}}_\mu\right)\,, \tag{36}$$

providing another contact current that is a part of the full contact term

$$j^a_{5\mu}(c) \,=\, \frac{i}{2f_\pi}\varepsilon^{amn}\,\bar{u}(p_1')\left(\gamma_\mu - \frac{\kappa^V}{2M}\sigma_{\mu\nu}q_\nu\right)\tau^m\,u(p_1) \times \Delta^\pi_F(q^2)\,\bar{u}(p_2')\hat{\mathcal{O}}^\pi_2(q_2)\tau^n u(p_2) + (1 \leftrightarrow 2). \tag{37}$$

This current is required by the current algebra prediction for the weak pion production and it corresponds to the well known ρ-π current. It looks like a potential one, but it is not connected to the potential and it satisfies the PCAC equation

$$q_\mu j^a_{5\mu}(c) \,=\, \frac{i}{2f_\pi}\varepsilon^{amn}\,\bar{u}(p_1')\;\not{q}_2\tau^m\,u(p_1)\,\Delta^\pi_F(q^2)\,\bar{u}(p_2') \times \hat{\mathcal{O}}^\pi_2(q_2)\tau^n u(p_2) + (1 \leftrightarrow 2) \,\equiv\, i f_\pi\, m^a_\pi(c)\,. \tag{38}$$

The amplitude $m^a_\pi(c)$ is generated from the $NN\pi\pi$ term $\Delta\mathcal{L}_{NN\pi\pi} = (i/4f_\pi^2)\bar{N}\gamma_\mu\vec{\tau}N\cdot(\partial_\mu\vec{\pi}\times\vec{\pi})$.

Let us note that the contact term $j^a_{5\mu}(c)$, eq. (37), is present in the HBChPT currents [13] also.

In the next section, we show that the same potential current (21) can be derived also within the HBChPT scheme.

2.2 The weak axial pion pair term within the HBChPT formalism

We first derive the positive-frequency nucleon Born term for the weak pion production amplitude on the nucleon in the leading order. To this end, we start from the lowest-order HBChPT Lagrangian [11,12,21]

$$\mathcal{L}^{(1)}_{\pi N} \,=\, -\bar{\mathcal{N}}_v\left[\,iv\cdot D \,+\, g_A\,S_v\cdot u\,\right]\mathcal{N}_v\,, \tag{39}$$

where $\mathcal{N}_v$ is the velocity dependent light component of the nucleon field Ψ, introduced in the HBChPT and it is defined as

$$\mathcal{N}_v \,\equiv\, e^{-iMv\cdot x}P_{v+}\Psi\,. \tag{40}$$

Here the four-velocity v_μ has the properties $v^2 = -1$ and $v^0 \geq 1$ and the projection operator P_{v+} is defined as

$$P_{v+} = \frac{1 - i \not{v}}{2} . \qquad (41)$$

For a choice $v_\mu = p_\mu / M$ we have

$$P_{v+} = \frac{M - i \not{p}}{2M} . \qquad (42)$$

Taking into account only the weak axial external interaction, $a_\mu = \mathcal{A}^a_\mu \tau^a/2$, we obtain in the leading order

$$g_A S_v \cdot u \approx g_A \tau^a S_v \cdot \mathcal{A}^a_\mu - \frac{g_A}{f_\pi} S_{v,\mu} (\vec{\tau} \cdot \partial_\mu \vec{\pi}) . \qquad (43)$$

Then the amplitude, corresponding to fig. 1a reads

$$M^v_c = -i \frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \bar{u}'_v \, (S_v \cdot q_2) \frac{P_{v+}}{v \cdot K} (S_v \cdot \mathcal{A}^a) \, u_v . \qquad (44)$$

Here N' and N are the normalization factors. We use in M^v_c the choice

$$v_\mu = p_\mu / M , \qquad \vec{p} = \vec{P} , \qquad p_0 = E(\vec{P}) . \qquad (45)$$

With this choice, the decomposition of the four-vector P_μ is

$$P_\mu = p_\mu + K_\mu , \qquad (46)$$

so that the scalar product $v \cdot K$ in eq. (44) is given by

$$v \cdot K = -v_0 \, (P_0 - E(\vec{P})) . \qquad (47)$$

Then we can write

$$\bar{u}'_v \, (S_v \cdot q_2) = -\frac{1}{2} \bar{u}'_v \, \gamma_5 (i \not{q}_2 + q_2 \cdot v) = \bar{u}'_v \left[S_v \cdot q_2^{st} - \frac{1}{2} \gamma_5 \, (P_0 - E(\vec{P}))(\gamma_4 + v_0) \right] . \qquad (48)$$

Employing eqs. (47) and (48) in eq. (44), we obtain

$$M^v_c = -i \frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \bar{u}'_v \left(S_v \cdot q_2^{st} \right) \frac{P_{v+}}{v \cdot K} (S_v \cdot \mathcal{A}^a) \, u_v - i \frac{2g_A^2}{f_\pi} N' N \tau^b \frac{\tau^a}{2} \frac{1}{2v_0} \bar{u}'_v \, \gamma_5 \gamma_4 \, P_{v+} \, (S_v \cdot \mathcal{A}^a) \, u_v . \qquad (49)$$

For the amplitude, corresponding to fig. 1b, we have

$$M^v_d = -i \frac{2g_A^2}{f_\pi} N' N \frac{\tau^a}{2} \tau^b \, \bar{u}'_v \, (S_v \cdot \mathcal{A}^a) \frac{P_{v+}}{v \cdot K} \left(S_v \cdot q_2^{st} \right) u_v + i \frac{2g_A^2}{f_\pi} N' N \frac{\tau^a}{2} \tau^b \frac{1}{2v_0} \bar{u}'_v \, (S_v \cdot \mathcal{A}^a) \, P_{v+} \gamma_5 \gamma_4 \, u_v . \qquad (50)$$

In M^v_d, we use the choice

$$v_\mu = p_\mu / M , \qquad \vec{p} = \vec{Q} , \qquad p_0 = E(\vec{Q}) . \qquad (51)$$

With this choice, the decomposition of the four-vector Q_μ is

$$Q_\mu = p_\mu + K_\mu , \qquad (52)$$

from which it follows that

$$v \cdot K = -v_0 \, (Q_0 - E(\vec{Q})) . \qquad (53)$$

Summing up the partial results (49) and (50), we obtain

$$M^v_{c+d} = M^v_{c+d}(st) + \Delta M^v_{c+d} , \qquad (54)$$

where

$$M^v_{c+d}(st) = -i \frac{2g_A^2}{f_\pi} N' N \bar{u}'_v \left[\left(S_v \cdot q_2^{st} \right) \frac{P_{v+}(P)}{v \cdot K} (S_v \cdot \mathcal{A}^a) \times \tau^b \frac{\tau^a}{2} + (S_v \cdot \mathcal{A}^a) \frac{P_{v+}(Q)}{v \cdot K} \left(S_v \cdot q_2^{st} \right) \frac{\tau^a}{2} \tau^b \right] u_v , \qquad (55)$$

and

$$\Delta M^v_{c+d} = i \frac{g_A^2}{f_\pi} \frac{1}{v_0} N' N \bar{u}'_v \left[\gamma_4 \gamma_5 \, P_{v+}(P) \, (S_v \cdot \mathcal{A}^a) \times \tau^b \frac{\tau^a}{2} - (S_v \cdot \mathcal{A}^a) \, P_{v+}(Q) \, \gamma_4 \gamma_5 \frac{\tau^a}{2} \tau^b \right] u_v . \qquad (56)$$

In order to obtain the two-nucleon amplitude, one should attach the propagator of the intermediate meson and the πNN vertex of the second nucleon. According to the generalized Weinberg's counting rules [12], such an amplitude has $\nu = -1$, like the contact amplitude $j^a_{5\mu}(c)$, eq. (37). The amplitude, following from $M^v_{c+d}(st)$ is analogue to the positive-frequency part of the nucleon Born term $J^{a(+)}_{5\mu}(ps)$, obtained with the PS πNN coupling. In our opinion, only this part belongs to the class of reducible diagrams that are not included in the exchange currents, if the currents are used in conjunction with the Schrödinger equation and the static one-pion exchange potential. On the other hand, one can obtain from the interaction ΔM^v_{c+d} a contact amplitude $\Delta J^a_{c+d,\mu}$

$$\Delta J^a_{c+d,\,\mu} = -\frac{g_A^3}{f_\pi^2} \frac{1}{v_{10}} N'_{v_1} N_{v_1} \bar{u}'_{v_1} \left[\gamma_4 \gamma_5 \, P_{v_1+}(P) S_{v_1,\mu} \times \frac{1}{2}(a^+ - a^-) - S_{v_1,\mu} P_{v_1+}(Q) \, \gamma_4 \gamma_5 \, \frac{1}{2}(a^+ + a^-) \right] \times u_{v_1} \Delta^\pi_F(q_2^2) \, N'_{v_2} N_{v_2} \bar{u}'_{v_2} \, (S_{v_2} \cdot q_2) \, u_{v_2} , \qquad (57)$$

that is of the same form as $\Delta J^a_{5\mu}(pv)$ of eq. (20). Making the non-relativistic reduction, one obtains the contact term that is identical with the current $\Delta j^a_{5\mu}(pv)$ of eq. (21).

2.3 Comparison of the WAEC

Let us now compare the space component of the WAEC of the pion range derived in the standard nuclear physics approach [4,7,34], based on the chiral Lagrangians, with a similar component in the HBChPT approach [12,18] taken

Table 1. Cumulative contributions to the cross-section $\sigma_{\nu d}$ ($\times 10^{-42}$ cm^2) from the weak axial exchange currents for various neutrino energies are displayed. The cross-section, calculated from the sum of the impulse approximation current and of the Δ isobar excitation of the π and ρ ranges is presented in the row labelled as IA+$\Delta(\pi+\rho)$. Other contributions correspond to the ρ-π current and to the pion potential term. The cross-section in the n-th row is given by the contribution of all previous currents, the n-th current including. The number in the bracket is the ratio of the n-th cross-section to the cross-section in the row above.

E_ν [MeV]	5	10	15	20	101
IA+$\Delta(\pi+\rho)$	0.0977 (-)	1.126 (-)	3.405 (-)	6.935 (-)	158.5 (-)
+ρ-π	0.0986 (1.009)	1.137 (1.010)	3.443 (1.011)	7.016 (1.012)	161.1 (1.016)
+p(π)	0.0978 (0.992)	1.127 (0.991)	3.408 (0.990)	6.940 (0.989)	157.9 (0.980)

in the leading order. The sum of the currents of the standard approach is given by the contribution of the potential term as derived in sect. 2, of the $\Delta(1232)$ isobar excitation and of the ρ-π current,

$$\begin{aligned}\vec{j}^{\,a}_{5,\,\pi} = \frac{g_A}{2Mf_\pi^2}\Big\langle g_A^2\Big\{\Big(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\Big)^2\frac{2M}{9(M_\Delta-M)}\,\vec{q}_2 \\ +\frac{1}{4}[\vec{q}+i(\vec{\sigma}_1\times\vec{P}_1)]\Big\}\tau_2^{\,a}+\frac{1}{4}\Big\{\Big[g_A^2\Big(\frac{f_{\pi N\Delta}}{f_{\pi NN}}\Big)^2 \\ \times\frac{2M}{9(M_\Delta-M)}+(1+\kappa_\rho^V)\Big]\,i(\vec{\sigma}_1\times\vec{q}_2) \\ +[g_A^2-(1+\kappa_\rho^V)]\,i(\vec{\sigma}_1\times\vec{q})+(g_A^2-1)\vec{P}_1\Big\} \\ \times i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\rangle(\vec{\sigma}_2\cdot\vec{q}_2)\,\Delta_F^\pi(\vec{q}_2^{\,2})+(1\leftrightarrow 2).\quad(58)\end{aligned}$$

The contribution from the Δ isobar excitation is specified by the factor $(\frac{f_{\pi N\Delta}}{f_{\pi NN}})^2/(M_\Delta-M)$, where M_Δ is the mass of the Δ isobar and $f_{\pi N\Delta}$ is the $\pi N\Delta$ coupling. Other terms, containing g_A^2, are from the potential current. In deriving eq. (58), we put the strong form factors $F_{BNN}(\vec{q}_i^{\,2})=1$, $\Delta_F^\rho(\vec{q}_1^{\,2})=1/m_\rho^2$ and we used the Goldberger-Treiman and KSFR relations, $M|g_A|=gf_\pi$ and $2f_\pi^2g_\rho^2=m_\rho^2$, respectively.

The leading order HBChPT currents were compared [18] with the standard currents [8] that contain the pion pair term with the PS πNN coupling. In comparing, this current was omitted. The argument was that it corresponds to the PS πNN coupling that is not chiral.

Here we take for comparison the currents $\vec{A}^{\,a\,:\,\nu 3}_{12}(1\pi)$, [[18], (A5)], but with the potential current (21) added. In our notation

$$\begin{aligned}\vec{A}^{\,a\,:\,\nu3}_{12}(1\pi)=\frac{g_A}{2Mf_\pi^2}\Big\langle\{2\hat{c}_3\,\vec{q}_2+\frac{g_A^2}{4}[\vec{q}+i(\vec{\sigma}_1\times\vec{P}_1)]\}\,\tau_2^{\,a} \\ +\frac{1}{4}\{(4\hat{c}_4+1)\,i(\vec{\sigma}_1\times\vec{q}_2)+[g_A^2-1-c_6]\,i(\vec{\sigma}_1\times\vec{q}) \\ +(g_A^2-1)\vec{P}_1\}\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\rangle \\ \times(\vec{\sigma}_2\cdot\vec{q}_2)\,\Delta_F^\pi(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,.\quad(59)\end{aligned}$$

The currents $\vec{j}^{\,a}_{5,\,\pi}$ and $\vec{A}^{\,a\,:\,\nu3}_{12}(1\pi)$ have an identical structure. This was achieved by respecting the chiral invariance and solving the double-counting problem in conjunction with the Schrödinger equation. In our opinion, it is the current (59) that should be used in the nuclear physics calculations with the nuclear wave functions derived using the Schrödinger equation.

Let us also note here that the weak axial exchange current [9] can be used in conjunction with the equation of motion, the first Born iteration of which coincides with the positive-frequency part of the nucleon Born term, constructed with the PV πNN coupling. In order to apply it in conjunction with the Schrödinger equation, one should remove the reducible piece from the positive-frequency part of the nucleon Born term and add the rest to the already derived exchange current [9]. The resulting current will be of the order $\mathcal{O}(1/M^3)$. If the pion exchange current [9] is constructed with the PS πNN coupling, according to the discussion after eq. (26), one should sum up the PCAC constraint term and the negative-frequency Born term (the pair term), both with the potential embedded. The resulting potential current will be the same as in the PV πNN coupling case.

It follows also from the discussion after eq. (26) that one needs to add the PCAC constraint term to the pair term [8,17], in order to obtain the chiral potential current.

For the numerical estimate of the discussed effect, we compute the contribution of the potential current to the cross-section for the low-energy electron neutrino-deuteron inelastic scattering in the neutral current channel,

$$\nu_e+d\rightarrow\nu_e'+p+n\,.\quad(60)$$

This reaction is important for studying the solar neutrino oscillations and it has been intensively studied both theoretically [15,33,35–38] and experimentally [39,40].

The model axial current considered contains the one-nucleon current and the WAEC (58), to which we add also the contribution from the Δ isobar excitation of the ρ range. Referring to sect. 4 of ref. [37] for the details, we present the results in table 1. The nuclear wave functions are generated by solving the Schrödinger equation with the Nijmegen I potential [41] and the transition $^3S_1$-$^3D_1\rightarrow{}^1S_0$ was considered . The used weak interaction constants are $G_F=1.1803\times10^{-5}$ GeV^{-2}, and $g_A=-1.267$.
It is seen from table 1 that the effects of the ρ-π and potential terms are $\sim 1\%$ and they cancel each other to a large extent. Since the total effect from the space part of WAEC is at the level of a few percent, it is important to correctly identify all the components of the WAEC that satisfy the PCAC and contribute sensibly.

3 Results and conclusions

The question of the interplay of the chiral invariance restriction and of the double-counting problem in the construction of the weak axial potential exchange currents of the pion range is discussed. It is shown that in order to avoid the double-counting problem, one should study the structure of the first Born iteration of the nuclear equation of motion and of the nucleon Born term. Only the part of the nucleon Born term, that is not contained in the first Born iteration contributes to the exchange currents. This current was constructed in conjunction with the Schrödinger equation in sect. 2. Then it is shown that the total potential exchange current, with the pion pair term included, satisfies the PCAC constraint (7). The construction is done in the leading order both in the chiral perturbation theory and in the HBChPT approach. The resulting potential term is the same in both approaches and it coincides with the potential term derived earlier from the hard pion Lagrangians. It is also shown that with the correct potential term taken into account, the leading order part of the space component of the long-range weak axial exchange currents of the HBChPT approach is identical with such a component obtained within the standard nuclear physics approach based on chiral Lagrangians. The same is also true for the pion exchange currents constructed in refs. [8,9].

Numerically, the contribution of the potential term is at the same level as the contribution from the well-known ρ-π current and the two contributions tend to cancel each other at low energies.

Let us note that in ref. [42], the time component of the electromagnetic exchange currents of the pion range was constructed in conjunction with the Blankenbecler-Sugar-Logunov-Tavkhelidze equation [43,44], that is a 3-dimensional reduction of the Bethe-Salpeter equation[4]. It was shown [20] that the resulting exchange charge density is equivalent to that obtained by such standard methods as are the transformation method [20] and the extended S-matrix method [16] and that it is independent of the form of the πNN coupling. This result provides a strong argument that the WAEC of the pion range constructed here, and more generally, the one-boson WAEC constructed in ref. [37] from chiral Lagrangians, can be used in standard nuclear physics calculations also in conjunction with the corresponding Lippmann-Schwinger equation, obtained [46] by the above-discussed reduction of the Bethe-Salpeter equation, and using the Bonn potentials [46–48] for generating the nuclear wave functions.

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[4] See also ref. [45].

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